

【연구논문】

Strategic Challengers and the Incumbency Advantage

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1. Introduction

As elections are the essential mechanism through which ordinary citizens play their roles in choosing the right representatives, whether regular elections could motivate politicians to represent voters' preferences has been studied in research on models of "electoral accountability" (Ashworth 2005, Austen-Smith and Banks 1989, Duggan 2000, Banks and Sundaram 1993, 1998, Fearon 1999, Ferejohn 1986, Reed 1994). Departing from Downs' electoral competition, these studies drop the commitment assumption and develop dynamic models in which politicians can choose a policy freely after being elected. Especially, Duggan (2000) constructs an infinitely repeated election model with one-dimensional policies and heterogenous voters, and proves the existence of equilibria where the median voter is decisive in every election and the policy outcome eventually lies in an interval around the median ideal policy.¹⁾

However, one drawback of the existing models of accountability is that they ignore the role of electoral campaigns. In those models, voters formulate expectations about future policy choices by an incumbent based on her past policy choice. By contrast, a challenger is randomly selected and voters know nothing but the distribution from which the challenger is drawn. The assumption about incumbents may be theoretically sound because the past records of incumbents are crucial to voters' judgments about them. The assumption about challengers, however, can be justified only when electoral campaigns are completely uninformative, so that voters cannot differentiate characteristics of challengers. However, not all types of campaign activities can be regarded as a mere cheap talk. Some efforts during electoral campaigns are costly and hence may provide useful information about challengers. For instance, the fact that a challenger builds a large amount of campaign money could be an indicator of her competence. If so, challengers may try to raise money in order to send a signal to voters that they are competent. This strategic interaction between a challenger and voters is an important component of electoral processes, and thus it must be taken

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- 1) There are a couple of extensions of the repeated election model with different contributions (Banks and Duggan 2002, Bernhardt, Dubey, and Hughson 2004, Bernhardt, Campuzano, Squintani, and C'amara 2009). Banks and Duggan consider a repeated election model with a general finite dimensional Euclidean space and show that Duggan's (2000) result is extended to a multidimensional policy space. Bernhardt, Dubey, and Hughson model repeated elections with term limits and show that the presence of term limits reduces the set of incumbents' policy choices that guarantee their reelection. By introducing political parties into Duggan's model, Bernhardt, et al. (2009) show that competition between two polarizing parties may result in more centralized policy outcomes.

into account.

We develop a game-theoretic model of infinitely repeated elections in which strategic challengers are present. The specific features of the model are as follows. There is a continuum of voters who have single-peaked preferences over one-dimensional policies. Politicians' characteristics are of two dimensions, policy preference and quality, which are private information. While voters have heterogeneous preferences over policies, they unanimously agree that high quality is better than low quality. In each period, a challenger is randomly selected from the pool of citizens and strategically decides the level of a costly signal that she will send to voters. An election between the challenger and the incumbent (the winner in the previous period election) is held, and the election outcome is determined by majority rule. In order to compare the challenger and the incumbent, voters use information from the challenger's signaling strategy as well as from the incumbent's past policy choice. The winner of the election becomes the incumbent and decides the policy outcome in the current period. This process is repeated infinitely. The model is dynamic, and therefore our analysis makes predictions about policy choices and electoral outcomes over time.

By a signal, in this paper, we mean any activity on the candidates' part satisfying two properties. First, a higher level of activity should cost more. Second, the cost for sending any given signal is at least as cheap for the higher quality candidate as for the lower quality candidate. As mentioned above, for example, the amount of campaign money could work as one of the most effective signals. Obviously raising more money should be more difficult for each candidate.

However, since campaign donors consider the quality of the candidate, it should not be harder for high quality candidates to collect the same amount of money than for low quality candidates. Also, previous office-holding experience can be another example of a costly signal. Empirically, previous office-holding experience works as the best for gauging challengers' quality, as much research on U.S. electoral competition demonstrates.

We prove that, under certain conditions, there exists an equilibrium with the following characteristics. First, we observe a positive signal from challengers only when the performance of incumbents is neither "too good" nor "too bad" with respect to the median voter's utility. When there is a positive signal in equilibrium, the challenger is elected. When the performance of an incumbent is too good (a centrist policy choice from a "high quality" incumbent), the incumbent is elected without being contested by a costly campaign from a challenger. When the performance is too bad (either an extreme policy choice from a high quality incumbent, or any choice from a low quality incumbent), a challenger can defeat the incumbent even without engaging in a costly campaign. Second, the signals from challengers convey information about their quality but no information about their policy preferences. Thus, in equilibrium, a positive signal is sent only by high quality challengers and by all types of high quality challengers.

Our findings contribute to the formal literature of electoral competition but also to our understanding of one of the most well-known topics in the American politics literature, the incumbency advantage. As a general rule in U.S. elections at both the national

and local levels, more incumbents secure their seats than lose them, and they have better chance of being reelected today than they had in the past (Mayhew 1974, Jewell and Breaux 1988, Cox and Morgenstern 1993, 1995, Cox and Katz 1996, Ansolabehere and Snyder 2004, Carson, Sievert, and Williamson 2015). In the 1960s and 1970s, only about 6-7 percent of incumbent members of Congress who had sought reelection were defeated. In 2002 a record 99 percent of incumbents who had pursued reelections won their elections. Also, in state legislative elections, on average, over 97 percent of incumbent legislators won their reelections in recent years, while they had more than a 6 percent defeat rate when they sought reelection in the 1970s.²⁾

For the sources of incumbency advantage and the causes of its growth, it is generally believed that there are other factors than the policy issue which voters consider when they decide whom to vote for in elections. First, incumbent legislators have good resources they can use for constituency services. By using franking privileges, staff support, and their operating budgets, legislators can provide many services to their constituents and this gives them a big electoral advantage against their opponents (King 1991, Cox and Morgenstern 1993, 1995, Carey, Niemi, and Powell 2000). Second, generally incumbents collect more campaign money and run better campaigns than their challengers (Abramowitz 1991, Jacobson 1997, ch.3). Third, most of incumbents enjoy name recognition challengers do not benefit

2) Carson, Sievert, and Williamson (2015) show that incumbents have been advantaged even when they extended the investigation back to the antebellum era.

from (Jacobson 1981, Campbell 1983, Serra and Cover 1992). Since incumbents perform constituency services, manage better campaigns, and have more media coverage, they become familiar to voters and generally get good evaluations. Lastly, most incumbents can deter strong challengers from entering the elections (Krasno and Green 1988, Squire 1991, Cox and Morgenstern 1993, Cox and Katz 1996, Levitt and Wolfram 1997, Ban, Llaudet, and Snyder 2014). When incumbents are running for reelection, most of the time the strongest challengers give up running against them, afraid of being defeated. Moreover, by raising large sums of early money, incumbents scare strong potential opponents off, which allows them to go uncontested (Squire 1991). Banks and Kiewiet (1989) theoretically show that weak challengers are more likely to run against incumbents than strong challengers by developing a model incorporating a primary election.

In contrast to the previous studies, this paper provides an explanation of the incumbency advantage without assuming any *ex ante* asymmetry between an incumbent and a challenger. In equilibrium in our model, the probability that an incumbent is reelected increases over time and finally converges to one. Since we do not assume any resource disparity between incumbents and challengers, and since we do not assume increasing competence from political experience, our result can be thought of as an endogenous emergence of the incumbency advantage. The main reason is as follows. Since voters can oust bad incumbents from office, as competitive elections are repeated, the incumbents who have survived are the good ones. Thus, the probability of incumbents' being

reelected increases over time. Similar arguments are made in different repeated election models (Banks and Duggan 2002, Duggan 2000). However, in those models, challengers are not strategic, so the result is somewhat transparent because the value of challengers is fixed at its expectation. By contrast, our result shows the emergence of the incumbency allowing for strategic campaigns of challengers.

Our findings show that asymmetry of information about candidates may be an important source of the incumbency advantage. In equilibrium, while a centrist incumbent can convince voters that she is moderate by *implementing* a centrist policy, a centrist challenger cannot credibly distinguish herself from a extremist challenger. However, in equilibrium, a high quality challenger can distinguish herself from low quality challengers by sending a costly signal. Thus, our theory of the incumbency advantage is distinct from models that incorporate the valence issue in the Downsian model (Bernhardt and Ingberman 1985, Gloseclose 2001, 2007, Berger, Munger, and Potthoff 2000, Ansolabehere and Snyder 2000, Aragonés and Palfrey 2002). Most of those models assume that voters award a valence advantage to one of the candidates exogenously and implicitly assume that the advantaged candidate is the incumbent. By contrast, our explanation does not rely on the exogenous quality difference between candidates.

Most of all, our results predict a non-monotonic relationship between the performance of an incumbent and the strength of the campaigns from strategic challengers. High quality challengers engage in a strong campaign activity only when the performance of an incumbent is at a medium level, that is, only when they can win the

election by active campaigning. This result is consistent with findings of empirical studies on U.S. elections. When incumbents are not strongly favored by voters, they are more likely to face strong challengers and lose their seats (Campbell 1983, Jacobson 1989, Squire 1989, Lublin 1994). Also, our finding is consistent with the “deterrence” explanation of the incumbency advantage. As mentioned, most of the incumbents can deter strong challengers from entering elections, and therefore, the strongest challengers confront incumbents only when there is a good chance of victory.

The rest of the paper proceeds as follows. In the next section, we describe the model. Section 3 presents the formal results of the model and Section 4 provides observational implications of our equilibrium analysis. Finally in section 5 we conclude. We leave all the formal proofs of results in the Appendix.

2. The Model

Let $X = [-1, 1]$ be a set of policies and let N denote a continuum of citizens with its size being a unit mass. Each citizen is characterized by her ideal policy in X , say z . We assume z is distributed by a probability density function f on X . It is assumed that f is continuous, positive, i.e., $f(z) > 0$ for all $z \in X$, and symmetric, i.e., $f(z) = f(-z)$ for all $z \in X$. There are two politicians, an incumbent and a challenger who are selected from citizens. When a citizen is selected as a politician, she is assigned a level of political competence (quality) $q \in Q = \{q_H, q_L\}$ ($q_H > q_L$). For simplicity, we

normalize q_L to 0. Then, the value of q_H can be interpreted as the quality difference between a high quality and a low quality candidate. The probability that a politician is endowed with a “high” quality (q_H) is $\pi \in (0, 1)$ and the probability that she is endowed with a “low” quality (q_L) is $1 - \pi$. We assume quality and ideal points are distributed independently. To define payoffs later, we need to define (stage) utility functions for citizens. For every ideal point $z \in X$ and for every $(x, q) \in X \times Q$, let denote the utility that a citizen with ideal point z receives when a policy x is implemented by a politician with quality q . That is, every citizen wants to minimize the distance between an implemented policy and her ideal point and prefers a high quality politician to a low quality one.³⁾

$$u(x, q|z) = -(x - z)^2 + q$$

The game proceeds as follows. In period 0, an individual is randomly chosen as an incumbent and selects a policy $x^0 \in X$. Then in each period $t = 1, 2, \dots$, (1) a challenger who runs against the incumbent of time $t - 1$ is randomly drawn from N and is assigned quality q ; (2) after observing the incumbent’s policy choice and quality level, the challenger decides the level of a costly signal $s^t \in \mathbb{R}_+$. Citizens observe the signal by the challenger, but not her type; (3) an election is held, in which all citizens simultaneously cast their ballots between the incumbent from $t - 1$ and the challenger; if the proportion of citizens voting for the incumbent is at least one half,

3) In the terminology of Groseclose (2007), voters have an one and half dimensional preference.

the incumbent is reelected and becomes the incumbent in t . Otherwise, the challenger wins and becomes the incumbent in period t ; (4) the incumbent at t implements a policy $x^t \in X$. This chosen policy and her quality q^t are observed by all citizens, and the game moves to $t + 1$. The payoff for a voter i with an ideal point z is

$$(1 - \delta) \sum_{t=0}^{\infty} \delta^t [u(x^t, q^t | z)],$$

and the payoff for a politician i with z in this game is

$$(1 - \delta) \sum_{t=0}^{\infty} \delta^t [u(x^t, q^t | z) + I(t)\rho - J(t)c(s^t | q)],$$

where $I(t)$ is the indicator function taking a value one if i is the office holder at t and zero otherwise, and $J(t)$ is also the indicator function taking a value one if i is selected as a challenger at t and zero otherwise. The parameter $\rho \geq 0$ is a nonnegative benefit for holding an office, $\delta \in [0, 1)$ is a common discount factor, and $c(s^t | q)$ is the cost for sending a signal s^t . For every $q \in Q$, $c(\cdot | q)$ is continuous and strictly convex. Also, $c(\cdot | q)$ is strictly increasing, and for every $s \in \mathbb{R}_+$, $c(s | q_L) \geq c(s | q_H)$. Thus, for a given quality level, a higher signal is more costly than a lower one and for a given signal level, a challenger with low quality pays at least as a high cost as one with high quality does. We also normalize $c(0 | q_L) = c(0 | q_H) = 0$.

Our solution concept is *simple equilibrium*. A simple equilibrium is

a perfect Bayesian equilibrium in *simple strategies*. A profile of simple strategies satisfies the following. First, a challenger's signaling depends only on the incumbent's policy choice and quality level in the previous period. Second, when voting at period t , voters only consider the policy choice and quality level of the incumbent at $t - 1$ and a challenger's level of signal at t . Third, each politician's policy choice is stationary in that whenever she is elected as an officeholder, she chooses the same policy determined by her type. Lastly, we consider only symmetric equilibria where individuals of the same type adopt the same strategies. If two citizens share the same policy preference, their voting strategies are the same and if two politicians have the same type (both policy preferences and quality level), their strategies for policy choice and signaling rule are the same.

From the last type-symmetry condition, we can express a simple strategy as only depending on individual types. Then, a voting rule is a function

$$v : X \times Q \times \mathbb{R}_+ \times X \rightarrow \{0, 1\},$$

where 0 means voting for a challenger and 1 means voting for an incumbent. That is, $v(x, q, s|z)$ is the ballot a voter with ideal point z casts after observing an incumbent's policy choice x and quality level q , and a challenger's signal s . A signaling strategy is a function

$$l : X \times Q \times X \times Q \rightarrow \mathbb{R}_+,$$

when, $l(x, q|z, q_C)$ is a level of a signal a challenger with an ideal policy position z and quality q_C sends when observing incumbent's x and q . Lastly, policy choice strategy is a function

$$p : X \times Q \rightarrow X,$$

where $p(z, q) \in X$ is the policy a politician with type (z, q) would choose whenever she holds the office.

Voters have beliefs on candidates' types after they observe the incumbent's policy choice and quality level, (x, q) , and a challenger's signal s . Let $\eta_I(\cdot | x, q)$ be the voters' belief about the incumbent's type conditional on observing policy choice x by an incumbent with quality q . Let $\eta_C(\cdot | x, q, s)$ be the voters' belief about a challenger's policy preference after observing an incumbent with policy choice x and quality q and a challenger's signal s .⁴⁾ Let $\tilde{\pi}(x, q, s)$ be the voters' beliefs of the probability that the challenger's quality is high after observing the same event. As is standard, all beliefs are updated using the Bayes rule whenever possible. Let $\sigma = (v, l, p, \eta_I, \eta_C, \tilde{\pi})$ denote a profile of simple strategies (and beliefs) of this game.

We now discuss the optimality conditions for each part of an equilibrium strategy profile. We first consider voting strategies. Since there is a continuum of voters, no one would be pivotal, which makes every voting behavior a trivial best response. Therefore, we revise our optimality condition in a standard way by assuming that citizens decide whom to vote for based on which candidate gives

4) Technically $\eta_I(\cdot | x, q)$ and $\eta_C(\cdot | x, q, s)$ are probability measures on X .

them the higher expected payoff. In other words, each voter behaves as if she was pivotal, i.e., she votes for a challenger if the expected payoff from electing a challenger is higher than the expected payoff from reelecting an incumbent. The expected payoffs are computed precisely in the appendix. For a given strategy profile $\sigma = (v, l, p, \eta_I, \eta_C, \tilde{\pi})$, for $b \in \{0, 1\}$, we denote the expected payoff for a voter with ideal point z from outcome b when she observes the incumbent's policy choice x and quality q and a signal s by $U_z(b|x, q, s; \sigma)$, where $b = 0$ denotes a win for the challenger while 1 a win for the incumbent. If σ is an equilibrium, then, it must satisfy the following condition: for all x, q, s , and z ,

$$v(x, q, s|z) = 1 \text{ iff } U_z(1|x, q, s; \sigma) \geq U_z(0|x, q, s; \sigma).$$

Note that this condition implies that when a voter is indifferent between the incumbent and a challenger, she votes for the incumbent.

Next we consider signaling choices of challengers. Given a strategy profile σ , for any $s \in \mathbb{R}_+$ and any $(t, q_C) \in X \times Q$, let $V_{(t, q_C)}(s|x, q; \sigma)$ denote the expected payoff for a challenger of type (t, q_C) from sending a signal s when observing an incumbent with (x, q) . In equilibrium,

$$\text{for all } t, q_C, x, q, s', \quad V_{(t, q_C)}(l(x, q|t, q_C)|x, q; \sigma) \geq V_{(t, q_C)}(s'|x, q; \sigma).$$

In equilibrium, for each given (x, q) , there exists cutpoint level of a signal $\underline{s}(x, q) \in \mathbb{R}_+$ such that every challenger, after observing an incumbent with policy choice x and quality q , chooses a level of

signal either $\underline{s}(x, q)$ or 0. To see this, suppose $\underline{s}(x, q)$ is the minimum signal level that induces a challenger's election given voters' strategies after observing (x, q) . If a challenger sends a signal s with $0 < s < \underline{s}(x, q)$, she still loses the election and pays a positive cost for signaling. The similar logic applies to the case where challenger sends a signal $s > \underline{s}(x, q)$ because in this case she is elected against incumbent but she pays more than it is needed. If there is no minimum level of signal, we can set $\underline{s}(x, q) = 0$. Therefore, when sending a signal, a challenger only considers either sending $\underline{s}(x, q)$ or sending no signal. Then, we can simplify the optimality condition of challenger's signaling strategy such that in equilibrium,

$$l(x, q|t, q_C) = \begin{cases} \underline{s}(x, q) & \text{if } V_{(t, q_C)}(\underline{s}(x, q)|x, q; \sigma) \geq V_{(t, q_C)}(0|x, q; \sigma), \\ 0 & \text{otherwise.} \end{cases}$$

Finally, we discuss policy choices for incumbents. Given a strategy profile σ , let $W_{(z, q)}(x; \sigma)$ be the expected payoff an incumbent with ideal point z and quality q receives when she implements a policy x . In equilibrium every policy choice must be optimal, that is,

$$\text{for all } z, q, x, \quad W_{(z, q)}(p(z, q); \sigma) \geq W_{(z, q)}(x; \sigma).$$

3. Formal Results

We now give the technical results of our election game. We begin

with a helpful lemma on median decisiveness and then present our existence theorem. The implication of our results will be discussed in the next section.

In the following Lemma, we identify an important necessary condition for a strategy profile to be a simple equilibrium. Note that the assumption that the distribution of citizen ideal points, f , is symmetric implies that zero is the unique median ideal point. Our first result establishes decisiveness of the median voter in elections.

Lemma 1 *For every equilibrium σ and every $(x, q, s) \in X \times Q \times \mathbb{R}_+$, an incumbent is elected if and only if $v(x, q, s|0) = 1$.*

In words, Lemma 1 states that, in every equilibrium and in every election, the winner is determined by the median voter's voting decision. That is, if the median voter votes for an incumbent, then the incumbent is reelected; otherwise, a challenger is elected. Thus, if the policy is one-dimensional and voters unanimously agree that what is a good quality of office holders, those who pursue to be elected must appeal to the voter who takes the median position with respect to the policy dimension.

The lemma establishes the robustness of Black's (1958) median voter theorem to dynamic interactions and 'one and half' dimension of policy spaces. Duggan (2000) proves that electoral outcomes are determined by the median voter's vote in his one-dimensional model of repeated election, so first shows that the well-known median voter theorem is extended to a dynamic electoral competition. Groschke (2007) proves that, in a static social choice problem where voters' utilities are determined by the one-dimensional policy and the office-holder's quality, the majority preference is transitive and coincide

with the median voter's preference. Our lemma extends Duggan's result to the 'one and half' dimensional policy spaces and Grosz's result to a dynamic non-cooperative setting, and thus shows a stronger robustness of the median voter theorem.

Lemma 1 is a consequence of Lemma 2.1 of Banks and Duggan (2006a). In it, they prove that if voters' preferences over alternatives are quadratic, then the majority preference over lotteries on the alternative space is identical to the 'core' voter's preference over them. In the model of this paper, the expected payoff from every strategy profile at every history can be thought of as the expected utility from a 'continuation lottery,' which is a lottery on the space $X \times Q$.⁵⁾ Although voters do not have quadratic preferences on $X \times Q$ in our model, they have quadratic preferences on the policy space X . Moreover, since their preferences over Q is separately additive and unanimous, the result of Banks and Duggan is transparently extended to the median decisiveness of our model. For this reason, we do not provide a separate proof of Lemma 1.

We now state the main result of this study. Let ν denote the variance of the distribution f , i.e., $\nu = \int_{-1}^1 x^2 f(x) dx$. Recall that q_H denotes the quality level of a high quality type politician and π denotes the probability that a randomly selected politician is endowed with high quality. Our main result shows that, under a certain condition, there is a simple equilibrium with some interesting characteristics. In the following, we first formally state the main theorem of the paper, and then we discuss the intuition of it.

5) To see how to specify such a continuation lottery, see Banks and Duggan (2006a).

Theorem 1 Let $\pi q_H \geq \nu$ and $\delta \geq \frac{1+\nu+2\sqrt{q_H+\nu}}{(1-\pi)(1+\rho+\nu)}$. Then there exists a simple equilibrium σ such that σ satisfies the following for some $w_1, w_2, c_1, c_2 \in (0, 1]$ ($w_1 < w_2, w_1 \leq c_1, w_2 \leq c_2$) and $\underline{s} > 0$.

1. Low quality incumbents' policy choices

- For all $z \in X, p(z, q_L) = z$.

2. High quality incumbents' policy choices

- $p(z, q_H) = z$ if $z \in [-w_1, w_1] \cup [-w_2, -c_1] \cup (c_1, w_2] \cup [-1, -\max\{c_1, c_2\}) \cup (\max\{c_1, c_2\}, 1]$.
- $p(z, q_H) = w_1$ if $z \in (w_1, c_1]$.
- $p(z, q_H) = -w_1$ if $z \in [-c_1, -w_1]$.
- $p(z, q_H) = w_2$ if $z \in (\max\{c_1, w_2\}, c_2]$.
- $p(z, q_H) = -w_2$ if $z \in [-c_2, -\max\{c_1, w_2\})$.

3. Challengers' signaling strategies

- For all $(x, q, z) \in X \times Q \times X, l(x, q|z, q_L) = 0$.
- For all $(x, q, z) \in X \times Q \times X, l(x, q|z, q_H) = \underline{s}$ if and only if $x \in [-w_2, -w_1] \cup (w_1, w_2]$ and $q = q_H$; otherwise, $l(x, q|z, q_H) = 0$.

4. Election outcomes

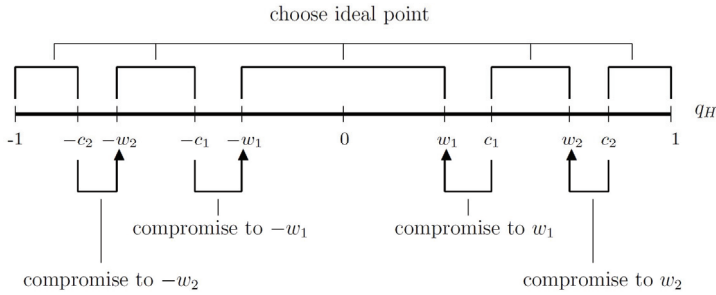
- If $x \in [-w_1, w_1]$ and $q = q_H$, then an incumbent is reelected.
- If $x \in [-w_2, -w_1] \cup (w_1, w_2], q = q_H$ and there is no positive signal from a challenger, then an incumbent is reelected.
- Otherwise, a challenger is elected.

In words, the policy choices of incumbents across different types are as follows. Here I just discuss positive types. The policy choices across negative types can be found in a symmetric way.

1. All low quality incumbents choose their own ideal policies and lose the election.
2. High quality incumbents whose ideal points are close to the median, $z \in [0, w_1]$, choose their ideal points and are reelected.
3. High quality incumbents whose ideal points are in $(w_1, c_1]$ choose the policy w_1 and are reelected.
4. High quality incumbents whose ideal points are in $(c_1, w_2]$ choose their ideal points and are reelected only with probability $1 - \pi$, that is, when a challenger is endowed with low quality.
5. High quality incumbents whose ideal points are in $(w_2, c_2]$ choose the policy w_2 and are reelected only with probability $1 - \pi$.
6. High quality incumbents whose ideal points are in $(c_2, 1]$ choose their ideal points and lose the election.

So, when high quality incumbents have policy preferences moderate enough, they implement policies which guarantee reelection. As the incumbents' preferences get further from the median ideal point, they choose intermediate policies to have only a positive probability of winning the election. When the high quality incumbents' policy preferences are extreme, however, they do not pursue winning the election and choose their own ideal points, and it is the same for all low quality incumbents. The policy choice strategies for high quality incumbents are visually expressed in Figure 1. In order to give the general intuition of the result, in this figure we present the case where $w_1 \leq c_1 \leq w_2 \leq c_2$.

Figure 1. High quality incumbents' policy choices



And, the characteristics of the equilibrium related to the electoral competition can be summarized as follows.⁶⁾

1. “Moderate” policy choices by an incumbent with high quality guarantee a reelection without being contested by a positive signal from a challenger.
2. If an incumbent with high quality chooses “intermediate” policies, she is reelected only when there is no positive signal from a challenger. If a high quality challenger is selected in this case, the challenger sends a signal and gets elected.
3. If an incumbent with high quality chooses “extreme” policies, she is ousted from the office without being contested by a positive signal from challenger.
4. All incumbents with low quality are ousted from the office without being contested by a positive signal from a challenger.

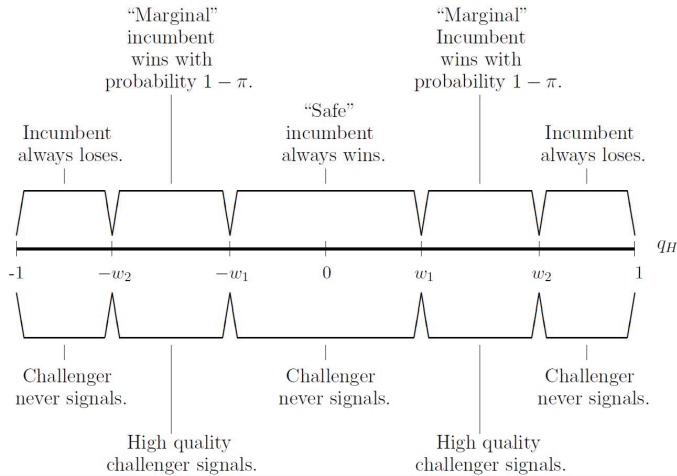
6) We discuss voters' belief about politicians' types on and off-the-equilibrium path in the Appendix.

5. Whenever a positive signal is sent on the equilibrium path, it is from a challenger with high quality and it conveys information only about challenger's quality.

That is, when the incumbents are of high quality and take centrist policies, they always win the election. Therefore, no challenger has an incentive to send a signal. When the incumbents choose extreme policies, they always lose. Again therefore, no challenger needs to send a signal. However, when the incumbents' policy choices are intermediate, high quality challengers send a signal but no low quality challengers do. In this case, the median voter prefers a high quality challenger to the incumbent, but prefers the incumbent to a low quality challenger. Therefore, the incumbent wins the election only when there is no signal. Finally, every low quality incumbent loses the election, so that there is no signal by any type of a challenger. The electoral consequences of incumbents' policy choices and challengers' signaling choices are summarized in Figure 2.

We now develop the intuition behind the main result. First, in the simple equilibrium there is an interval of policies, $[-w_1, w_1]$, such that incumbents' choosing policies in the interval always guarantees to win the next election. Why do some policy choices of incumbents with high quality guarantee reelection? Suppose an incumbent with high quality chooses the ideal policy of the median voter, zero. By stationarity of policy choices, voters expect that the incumbent would choose the same policy if reelected. Note that this outcome (zero policy and high quality) is the best for the median voter among all possible outcomes. Thus, unless the median voter is sure that a

Figure 2. Electoral outcomes with high quality incumbent as a function of incumbent's policy choice



challenger has high quality and will choose the zero policy, the median voter strictly prefers reelecting the incumbent to rejecting her. Suppose that a level of a signal can confirm that a challenger is of high quality and shares the preference with the median voter and that the median voter would vote for a challenger if the level of a signal is observed. Then, any challenger with extreme policy preferences would send the exactly same level of a signal, since the cost of doing so is constant across different policy preferences and ones with extreme preferences are more willing to change the current policy status quo. Thus, contradicting our supposition, no signal can inform voters that a challenger is of high quality and has the zero ideal point. Therefore, when observing zero policy choice from a high quality incumbent, the median voter strictly prefers the incumbent to any challenger. By continuity of utility functions, this argument must

hold for policies very close to the median, that is, $[-w_1, w_1]$.

What then determines the size of the interval, i.e., the magnitude of w_1 ? As discussed in the previous paragraph, a signal from challengers cannot reveal different policy preferences in our equilibrium. Thus, as long as a policy from the incumbent, say x , gives as a high payoff to the median voter as the expected payoff from electing a high quality challenger with an unknown policy preference, implementing x guarantees permanent reelection. So, w_1 is the policy from which the median receives the same payoff as the one expected from a high quality challenger.

We now discuss the case that incumbents choose a policy further from the median ideal point than w_1 . By symmetry of strategies, it is enough to discuss only positive policies. Suppose that an incumbent implements a policy x close to but greater than w_1 . In this case, every high quality challenger sends the same level of a positive signal, \underline{s} , but no low quality challenger does so in our equilibrium. Thus, if voters observe a signal from a challenger, they believe the challenger is of high quality; and otherwise, they believe she is of low quality. Since $x > w_1$ and the median voter is indifferent between electing an incumbent who chose w_1 and electing a high quality challenger, the median voter strictly prefers electing a high quality challenger to electing an incumbent who chose x . However, if x is close enough to w_1 , the median voter prefers electing the incumbent to a low quality challenger. Thus, we can find an interval $(w_1, w_2]$ such that, if the chosen policy lies in the interval, the election outcome depends on campaign activity from the challenger: if the challenger sends a signal at least as great as \underline{s} , the challenger

wins the election; otherwise, the incumbent wins. Finally, if an incumbent chooses too extreme policies, i.e., ones greater than w_2 , then since policies are too bad for the median voter, the incumbent will not be reelected regardless of the challenger's decision. The point w_2 is the policy such that the median voter is indifferent between electing a high quality incumbent who chose w_2 and electing a low quality challenger with an unknown policy preference.

The features of incumbents' policy choices in the equilibrium are all intuitive. First, given the voting strategy of the median voter discussed before, any low quality incumbent has no chance to be reelected regardless of her policy choice. Thus, she will choose the best policy for herself to maximize the current policy payoff. Now consider a high quality incumbent. By symmetry, discussing only positive ideal point cases causes no loss of generality. Assume the incumbent's ideal point is less than or equal to w_1 . Then by choosing her ideal point she can achieve both of her goals, i.e., to have the best policy and to remain in the office. Thus, the incumbent obviously will choose her ideal point.

By contrast, if the incumbent's ideal point is in $(w_1, w_2]$, she faces a trade-off. On the one hand, she can compromise to choose w_1 and can remain in the office in the future. On the other hand, she can implement her own ideal point to have the best policy in the current period and risks a positive probability of facing a high quality challenger and being ousted from the office in the next period. Therefore, if the expected utility of compromising to w_1 is greater than the expected utility of choosing her ideal point, then the incumbent will choose w_1 , and otherwise her choice will be her ideal

point. If the incumbent's ideal point is close to w_1 , her loss of the policy payoff by compromising is small, so the expected utility of compromising to w_1 would be greater than the expected utility of choosing her ideal point. By contrast, if it is far from w_1 , the inequality would be reversed. The point c_1 is such that the incumbent with ideal point c_1 is indifferent between implementing w_1 and implementing c_1 . From the above logic, if the incumbent's ideal point is less than c_1 , she will compromise to implement w_1 , and otherwise, she will choose her own ideal point.

High quality incumbents whose ideal points are in $(w_2, 1]$ face a similar trade-off. There are three different choices for them: to compromise to w_1 and stay in office for sure, to compromise to w_2 and win the election with a positive probability $1 - \pi$, and to choose their ideal points and lose the election for sure. When $c_1 < w_2$, the first option is not as good as the second option. So, if an incumbent's ideal point is close to w_2 , she will compromise to implement w_2 , otherwise, she will choose her ideal point. Similarly to c_1 , c_2 is the policy such that the incumbent with ideal point c_2 is indifferent between implementing c_2 and implementing w_2 . The mathematical characterizations of all four cutpoints, w_1 , c_1 , w_2 , and c_2 , are provided in the Appendix.

In Theorem 1, we present sufficient conditions for an equilibrium with the discussed characteristics to exist: $\pi q_H \geq \nu$ and $\delta \geq \frac{1+\nu+2\sqrt{q_H+\nu}}{(1-\pi)(1+\rho+\nu)}$. The former is derived from the condition that in equilibrium every low quality incumbent loses the election against an unknown challenger. That is, the median voter must prefer electing

an unknown challenger to the best policy (the median policy) from a low quality incumbent. Since we normalize the low quality to $q_L = 0$, q_H measures the quality difference between competent and incompetent politicians. Thus, the condition is met when the quality difference is great, the proportion of competent politician is high, or the variance of ideal points of politicians is small. The latter is derived from the condition that, when intermediate policies are chosen by high quality incumbents, every high quality challenger has the incentive to send a positive signal and no low quality challenger has the incentive. Notice that we do not impose a strong restriction of the cost of signaling. We allow the case that high quality challengers and low quality challengers share the same cost function. Finally, note that these conditions are only sufficient, but not necessary.

4. Observational Implications

In this section, we discuss implications of our equilibrium analysis, and then we present the dynamic properties and comparative statics of our simple equilibrium. Most notably, Theorem 1 predicts that strong campaign activity from challengers will be observed only when the performances of incumbents are at the medium level. Thanks to the median decisiveness given by Lemma 1, we may measure the performance of an incumbent by the utility of the median voter from the incumbent (with respect to both quality and a chosen policy) in the previous period. From this standard, incumbents with good

performance are those with high quality who chose a policy close to the median ideal point. In our equilibrium, every challenger gives up costly campaigning confronting such strong incumbents so that strong incumbents are reelected without seriously being challenged. By contrast, weak incumbents (those who chose a policy too far from the median or are incompetent in the valence dimension) will not be supported by the median voter, regardless of campaign from a challenger. Thus, in this case, no challenger has to launch a costly campaign since she can beat the incumbent without doing so. Therefore, we will observe strong campaign activity from challengers only in the case of incumbents with the medium strength, i.e., the case that effort is needed and can be effective.

The existing models of repeated elections provide no testable implication about the relationship between incumbents' performances and the level of campaign activities from challengers because the models do not include a strategic challenger. By contrast, we provide one. Specifically, the strength of incumbents and the campaign activities of challengers will have a non-monotonic (thus nonlinear) relationship. If we regress the level of campaign activities from challenger, for example, measured by campaign spending of strong challengers, on the strengths of incumbents, we will observe that the level of challenger's campaign activities increases initially in the low value range of the incumbent's strength and then it decreases in the high value range of the incumbent's strength. Thus, when empirically studying the relationship between these two variables, the usual linear statistical model will have a problem of misspecification.

In fact, we frequently observe that strong potential challengers give

up entering electoral competition, being afraid of being defeated by strong incumbents, which is consistent with our prediction. We may not often observe the cases that a challenger does not conduct electoral campaign actively when the incumbent is weak. The reason for this, however, may be that we observe a kind of censored data. Weak incumbents often decide not to run for reelection, so that such cases are not observed.

We now discuss the implications of our model on the incumbency advantage. First, in the equilibrium of our main result, an incumbent is reelected only when her performance is “not bad.” In other words, a low quality incumbent or a high quality incumbent with too extreme preference is not reelected in our equilibrium.

Second, some incumbents, however, may have some advantage because their political characteristics (valence and policy preferences) are better known to voters than those of challengers are. Consider an incumbent of high quality who chose a policy $x \in [-w_1, w_1]$. A selected challenger may be a better politician to the median voter than the incumbent, i.e., the challenger may be of high quality and have the ideal point closer to the median than x . However, there is no way that the challenger convince voters that she is a better candidate. This is true although we allow, in the model, that the challenger can send a costly signal. However, notice that this information asymmetry does not always favor incumbents. When incumbents are ‘bad,’ they will reveal it more easily than challengers.

One important question is whether our theory based on the assumption of rational voters can explain the huge incumbency advantage we observe in data. The next result is a positive answer to

this question.

Proposition 1 *The simple equilibria have the following dynamic properties:*

- *The ex ante probability that an incumbent gets reelected at t is increasing in t and converges to 1.*
- *The probability that a low quality politician holds office at t is decreasing in t and converges to 0.*

In our equilibrium, as time goes by, the probability that we observe an incumbent's reelection increases and finally converges to one. This prediction is consistent with observed data that exhibits a growing incumbency advantage over time. The main reason of this result is that if a politician is the incumbent in a later period, it is highly probable that she is selected by voters before. That suggests that she is a good politician for the median and so she will be reelected in later elections. Thus, dynamic interactions between voters and politicians under representative democracy work as an overtime selection process of good politicians. This result is in the same line of Zaller's (1998) electoral selection theory. He suggests that incumbents are better politicians than most of their opponents because candidates who have won in the past election must possess some characteristics that attract voters, and therefore, win in the future elections. That is, there are electoral selections through repeated elections.⁷⁾

7) There is increasing literature on the electoral selection. Ashworth (2005) models a repeated election game where voters reelect an incumbent

The boundary of the set of policies which guarantee incumbents' permanent winning in simple equilibria, w_1 , depends on the values of parameters of the game. So does the size of the set of politicians who compromise, c_1 . The next result establishes the comparative statics of simple equilibria.

Proposition 2 *In equilibrium, w_1 and c_1 satisfy the following properties:*

- w_1 is decreasing in δ .
- w_1 is decreasing in ρ .
- c_1 is increasing in δ .
- c_1 is increasing in ρ .

The results of the comparative statics are very intuitive. A higher δ means more patient voters. As the median voter becomes patient, she is willing to wait until she gets an office-holder who would implement a policy close enough to her ideal point. Then, even though the incumbent's policy choice is not very far from the median ideal point, she can be ousted and replaced with a challenger. Moreover, if δ is high, then politicians are also patient. Thus, instead of pursuing the current best policy, they are more inclined to compromise to implement a policy that guarantees a reelection. So,

considering her level of ability as well as her constituency service. The incumbency advantage arises since voters learn about the incumbent's ability and only reelect ones with high level of ability. That is, the distribution of incumbent's ability is better than the one of unknown challenger's, a selection effect. Gowrisankaran et al. (2003) empirically show the selection effect which explains the strong incumbency advantage in the U.S. Senate elections.

w_1 decreases and c_1 increases as δ increases.

A similar logic applies to the case about the office holding benefit ρ . If the office benefit increases, then more types of politicians pursue a reelection and so compromise. Then, since more politicians will compromise, the continuation value of challengers increases, implying that the median voter can use a higher standard for retaining an incumbent. Thus, w_1 decreases and c_1 increases as ρ increases.

In our game, the welfare of the median voter, measured by the ex ante equilibrium payoff, is a decreasing function of w_1 . Thus, as political actors become more patient and as the benefit of office increases, the median voter becomes better off. This can be a supportive argument of a more frequent election and a higher salary for representatives. Note that the existing models of repeated elections have results in a similar vein (e.g., Ferejohn 1986, Duggan 2000). Proposition 2 extends those results to the environment where there are strategic challengers.

Also, the result that as the office holding benefit, ρ , increases, c_1 increases makes interesting predictions of the effect of higher office holding benefit for representatives. Note that a bigger c_1 implies a larger proportion of incumbents who are reelected for sure and a smaller probability of elections being actively competitive (with challenger's signaling). Then, first, it predicts that higher benefit of office holding increases the incumbency advantage, and second, it predicts higher benefit of office holding decreases the level of electoral competitiveness. That is, for example, as the job of legislators becomes more valuable, the size of the incumbency

advantage gets larger and the election becomes less competitive. This prediction gets a good support from the literature on the state professionalization (Cox and Morgenstern 1993, 1995, Carey, Niemi, and Powell 2000, Weber, Tucker, and Brace 1991). Generally, it is shown that state legislative professionalization, which is measured by session length, salary, and legislators' operating resources, increases the incumbency advantage and decreases the electoral competitiveness. Cox and Morgenstern find that the amount of legislator's operating budget explains the difference in the size of the incumbency advantage between states, and Carey et al. show legislators' salaries play an important role in a bigger incumbency benefit in different states. About the decreasing electoral competitiveness in state legislative elections, Weber et al. show that the size of legislative expenditures is one of the most important factors to explain the tendency.

5. Discussion

In the simple equilibrium we found in this paper, once the incumbent implements a policy, voters know the incumbent's future policy choice due to the stationarity as well as her quality level. However, most of the time voters do not know about the challenger's type and even when they know the challenger's quality by observing her signal, they do not know her policy preference. As a result, when a high quality incumbent has centrist policy preference, she can be reelected against any challenger without being contested by strong

challengers.

However, our equilibrium shows that under a wide range of parameter values challengers' signaling conveys information about her quality level. When a high quality incumbent has an intermediate policy preference, a high quality challenger can be elected against the incumbent by sending a costly signal. In this case voters would elect the incumbent only when there is no signal. Thus, challengers' good quality and campaigning help their electoral success, which is well supported by Squire's (1992) empirical findings. Using 1988 U.S. Senate election data, he shows that when challengers have high quality and run campaigns well, they are better known and are more likely to get votes against incumbents. Our equilibrium shows that a strategic challenger's signaling reduces the benefit of incumbency, and when the incumbent's policy choice is not good enough, high quality challengers can win the election against the incumbent. This result is in the same line of the finding of Abramowitz (1975). According to him, the incumbency advantage comes from the reputation, which challengers cannot enjoy. However, the reputation is built based on the incumbent's performance in office and when her performance is bad, the incumbent cannot be supported in elections.

Also, we find that moderate incumbents take a policy closer to the median when there is a signaling in equilibrium comparing to ones which they would take if there is no signaling. Voters have higher expected payoffs from electing a challenger due to the strategic campaigning by competent candidates and it in turn makes them have a higher standard for reelecting incumbents. In other words, the presence of strong challengers and the possibility of the loss in

reelection when they are challenged by a strong candidate enforce some incumbents take more moderate policies in order to stay in their seats. In this sense, this paper provides a good support for the argument that competitive elections keep public officials more accountable.

Appendix

Proof of Theorem 1

Let Θ be the set of parameters that satisfies the supposition of the theorem; that is,

$$\Theta = \left\{ (\delta, \pi, q_H, \nu, \rho) \in [0, 1) \times (0, 1) \times \mathbb{R}_+ \times (0, 1) \times \mathbb{R}_+ \mid \pi q_H \leq \nu \text{ and } \delta \geq \frac{1 + \nu + 2\sqrt{q_H + \nu}}{(1 - \pi)(1 + \rho + \nu)} \right\}.$$

Let $\theta = (\delta, \pi, q_H, \nu, \rho) \in \Theta$. Let $w_2(\theta) = \sqrt{q_H + \nu}$, $\alpha(\theta) = \frac{1-\delta}{1-\delta(1-\pi)}$ and $\beta(\theta) = (1 - \pi)\alpha(\theta)$.

Define the function $\gamma : \mathbb{R}_+ \times \Theta \rightarrow \mathbb{R}$ so that, for each $(z, \theta) \in \mathbb{R}_+ \times \Theta$,

$$\gamma(z|\theta) = -(1 - \delta)z^2 + 2\alpha(\theta)w_2(\theta)z - \alpha(\theta)w_2(\theta)^2 + \delta\beta(\theta)[w_2(\theta)^2 + \rho].$$

Note that, for each given θ , $\gamma(\cdot|\theta)$ is strictly decreasing on the right of $w_2(\theta)$ and $\gamma(w_2(\theta)|\theta) \geq 0$. Thus, there exists a unique z such that $z \geq w_2(\theta)$ and $\gamma(z|\theta) = 0$. For each $\theta \in \Theta$, let $\hat{c}_2(\theta)$ be the unique point such that $\gamma(\hat{c}_2(\theta)|\theta) = 0$ and $\hat{c}_2(\theta) \geq w_2(\theta)$. Define the function $\psi : \mathbb{R}_+ \times [0, 1] \times \Theta \rightarrow \mathbb{R}$ so that, for each $(z, x, \theta) \in \mathbb{R}_+ \times [0, 1] \times \Theta$,

$$\psi(z, x|\theta) = -\alpha(\theta)z^2 + 2xz - \alpha(\theta)x^2 + [1 - \alpha(\theta)]\rho.$$

For each $(x, \theta) \in [0, 1] \times \Theta$, $\psi(\cdot, x|\theta)$ is strictly decreasing on the right of x and $\psi(x, x|\theta) \geq 0$. Let $\xi(x|\theta)$ be the unique point such that $\psi(\xi(x|\theta), x|\theta) = 0$ and $\xi(x|\theta) \geq x$. Define the function $\eta : \mathbb{R}_+ \times [0, 1] \times \Theta \rightarrow \mathbb{R}$ so that, for each $(z, x, \theta) \in \mathbb{R}_+ \times [0, 1] \times \Theta$,

$$\eta(z, x|\theta) = 2[x - \alpha(\theta)w_2(\theta)]z + [1 - \alpha(\theta)]\rho + \alpha(\theta)[w_2(\theta)^2 - x^2].$$

For each $\theta \in \Theta$ and each $x \in [0, \alpha(\theta)w_2(\theta))$, let $\zeta(x|\theta)$ be the unique point such that $\eta(\zeta(x|\theta), x|\theta) = 0$. Define the function $\mu : \mathbb{R}_+ \times [0, 1] \times \Theta \rightarrow \mathbb{R}$ so that, for each $(z, x, \theta) \in \mathbb{R}_+ \times [0, 1] \times \Theta$,

$$\mu(z, x|\theta) = -(1 - \delta)z^2 + 2xz + \delta\rho + \delta\beta(\theta)w_2(\theta)^2.$$

Note that $\mu(\cdot, x|\theta)$ has a unique critical point greater than x and $\mu(x, x, \theta) > 0$. For each $(x, \theta) \in [0, 1] \times \Theta$, let $\phi(x|\theta)$ be the unique point such that $\mu(\phi(x|\theta), x|\theta) = 0$ and $\phi(x|\theta) > x$.

Define the function $\hat{c}_1 : [0, 1] \times \Theta \rightarrow \mathbb{R}_+$ so that, for each $(x, \theta) \in [0, 1] \times \Theta$,

$$\hat{c}_1(x|\theta) = \begin{cases} \xi(x|\theta) & \text{if } \xi(x|\theta) \leq w_2(\theta), \\ \zeta(x|\theta) & \text{if } \xi(x|\theta) > w_2(\theta) \text{ and } \zeta(x|\theta) \leq \hat{c}_2(\theta), \\ \phi(x|\theta) & \text{otherwise.} \end{cases}$$

Let $\bar{c}_1(x|\theta) = \min\{\hat{c}_1(x|\theta), 1\}$, $\bar{w}_2(x|\theta) = \min\{\max\{\hat{c}_1(x|\theta), w_2(x|\theta)\}, 1\}$, and $\bar{c}_2(x|\theta) = \min\{\max\{\hat{c}_1(x|\theta), \hat{c}_2(x|\theta)\}, 1\}$.

Define the function $U : [0, 1] \times \Theta \rightarrow \mathbb{R}$ so that, for each $(x, \theta) \in [0, 1] \times \Theta$, $U(x|\theta)$ satisfies the following equality:

$$\begin{aligned} U(x|\theta) = & 2 \left[\int_0^x (-y^2 + q_H) f(y) dy + \int_x^{\bar{c}_1(x|\theta)} (-x^2 + q_H) f(y) dy \right. \\ & + \int_{\bar{c}_1(x|\theta)}^{\bar{w}_2(x|\theta)} [\alpha(\theta)(-y^2 + q_H) + (1 - \alpha(\theta))U(x|\theta)] f(y) dy \\ & + \int_{\bar{w}_2(x|\theta)}^{\bar{c}_2(x|\theta)} [\alpha(\theta)(-w_2(\theta)^2 + q_H) + (1 - \alpha(\theta))U(x|\theta)] f(y) dy \\ & \left. + \int_{\bar{c}_2(x|\theta)}^1 ((1 - \delta)(-y^2 + q_H) + \delta[-\beta(\theta)\nu + (1 - \beta(\theta))U(x|\theta)]) f(y) dy \right]. \end{aligned}$$

Note that, for all $x \in [0, 1]$, $U(x|\theta) \leq q_H$. For each given $\theta \in \Theta$, define the function $K(\cdot|\theta) : [0, 1] \rightarrow \mathbb{R}$ so that, for each $x \in [0, 1]$,

$$K(x|\theta) = q_H - U(x|\theta) - x^2.$$

Lemma 2 Let $r(\theta) = \sqrt{q_H - \frac{(1-\pi)(1-\delta)}{\pi}\nu}$. Then, for all $\theta \in \Theta$, $K(r(\theta)|\theta) \leq 0$.

Proof: The lemma can be proven by showing that, first, $K(r(\theta)|\theta) \leq 0$ when $\delta = 0$ and, second, $K(r(\theta)|\theta)$ is decreasing in δ . The detailed proof will be provided upon request.

It is obvious that $K(0|\theta) \geq 0$. By Lemma ??, $K(r(\theta)|\theta) \leq 0$. Thus, there exists a point $x \in [0, r(\theta)]$ such that $K(x|\theta) = 0$. Fix θ . Let w_1 be the point such that $K(w_1|\theta) = 0$. Let $w_2 = \bar{w}_2(w_1|\theta)$, $c_1 = \bar{c}_1(w_1|\theta)$, and $c_2 = \bar{c}_2(w_1|\theta)$. Since $\delta \leq \frac{1+\nu+2\sqrt{q_H+\nu}}{(1-\pi)(1+\rho+\nu)}$, there exists $\underline{s} \in \mathbb{R}_+$ such that $1 + \rho + \nu + \frac{2\sqrt{q_H+\nu}}{1-\delta(1-\pi)} \leq c(\underline{s}|q_L)$ and $c(\underline{s}|q_H) \leq \frac{\rho}{1-\delta(1-\pi)}$.

Define a profile of strategies and beliefs $\sigma = (v, l, p, \eta_I, \eta_C, \tilde{\pi})$ as follows. For simplicity, we only specify the median voters' voting strategy.

- $v(x, q, s|0) = 1$ if and only if

$$q = q_H \wedge \left[x \in [-w_1, w_1] \vee (x \in [-w_2, -w_1) \cup (w_1, w_2] \wedge s < \underline{s}) \right].$$

- l and p are as defined in the theorem.
- Beliefs η_I , η_C , and $\tilde{\pi}$ are derived from v, l, p via Bayes rule on the equilibrium path.

Off the equilibrium path:

1. Once a politician chooses a policy that is not supposed to be adopted, voters believe that he/she will choose the same policy in the future.
2. No policy deviation from an incumbent affects voters' belief about challengers.
3. If a challenger sends a signal when no signal is supposed to be observed, then voters always believe that the challenger is of high quality.

It is clear that σ satisfies the properties stated in the theorem. We now need to show that σ is an equilibrium. Let U_z^H denote the expected utility for voters with ideal point z from electing a high quality challenger with unknown policy preference. Similarly, U_z^L denotes the expected utility from electing a low quality challenger and \bar{U}_z denotes the expected utility from electing a challenger with unknown quality. Notice that U_z^H can be understood as the expected utility from a lottery. Since every voter has quadratic preference over policies and policy choices of politicians are symmetric around zero, it must be the case that $U_z^H = u(0; z) + U_0^H$. Note that, from the specification of σ ,

$$U_z^H = U(w_1|\theta) = -w_1^2 + q_H,$$

where the second equality uses the fact that $K(w_1|\theta) = 0$. Also,

$$\begin{aligned} U_z^L &= (1 - \delta) \int_{-1}^1 [u(y; z) + q_L] f(y) dy + \delta \bar{U}_z \\ &= (1 - \delta) [u(0; z) - \nu] + \delta \bar{U}_z, \end{aligned} \tag{1}$$

and

$$\bar{U}_z = \pi U_z^H + (1 - \pi) U_z^L. \quad (2)$$

Using (1) and (2), we obtain

$$U_z^L = \alpha(\theta)[u(0; z) - \nu] + [1 - \alpha(\theta)]U_z^H,$$

and

$$\bar{U}_z = u(0; z) - \beta(\theta)\nu + [1 - \beta(\theta)]U_z^H.$$

Consider optimality of the voting strategy for the median voter. Let $x \in [-w_1, w_1]$ and $s \in \mathbb{R}_+$. Then

$$U_0(1|x, q_H, s; \sigma) = -x^2 + q_H \geq -w_1^2 + q_H = U_0^H = U_0(0|x, q_H, s; \sigma).$$

Thus, $v(x, q_H, s|0) = 1$ is optimal. Let $x \in [-w_2, -w_1) \cup (w_1, w_2]$. Suppose $s \geq \underline{s}$. Then

$$U_0(1|x, q_H, s; \sigma) = \alpha(\theta)[-x^2 + q_H] + [1 - \alpha(\theta)]U_0^H \leq U_0^H = U_0(0|x, q_H, s; \sigma).$$

So $v(x, q_H, s|0) = 0$ is optimal. Now suppose $s < \underline{s}$. Then

$$U_0(1|x, q_H, s; \sigma) \geq \alpha(\theta)[-w_1^2 + q_H] + [1 - \alpha(\theta)]U_0^H = U_0^L = U_0(0|x, q_H, s; \sigma).$$

Thus, $v(x, q_H, s|0) = 1$ is optimal. Let $x \in [-1, -w_2) \cup (w_2, 1]$. Then

$$U_0(1|x, q_H, s; \sigma) \leq U_0^L \leq \bar{U}_0 \leq U_0^H.$$

So, $v(x, q_H, s|0) = 0$ is optimal for all s . Finally, consider the case where the incumbent is of low quality. For every x , $U_0(1|x, q_L, s; \sigma) \leq U_0(1|0, q_L, s; \sigma) = \delta \bar{U}_0$. Note that, since $w_1 \leq r(\theta)$,

$$\bar{U}_0 = -\beta(\theta)\nu + [1 - \beta(\theta)](-w_1^2 + q_H) \geq 0.$$

Thus,

$$U_0(0|0, q_L, s; \sigma) = \bar{U}_0 \geq \delta \bar{U}_0 \geq U_0(1|x, q_L, s; \sigma).$$

Therefore, it is optimal not to elect any incumbent with low quality.

Now consider signaling strategies. It is obvious that challengers send zero signal when signaling does not affect the election outcome. So, it is enough to consider that case that $x \in [-w_2, -w_1] \cup (w_1, w_2]$. Take any ideal point z .

$$\begin{aligned} V_{(z,q_H)}(\underline{s}; x, q_H) - V_{(z,q_H)}(0; x, q_H) &\geq V_{(x,q_H)}(\underline{s}; x, q_H) - V_{(x,q_H)}(0; x, q_H) \\ &= (1 - \delta) \left[\frac{\rho}{1 - \delta(1 - \pi)} - c(\underline{s}|q_H) \right] \geq 0. \end{aligned}$$

$$\begin{aligned} V_{(z,q_L)}(\underline{s}; x, q_H) - V_{(z,q_L)}(0; x, q_H) &\leq V_{(-1,q_L)}(\underline{s}; w_2, q_H) - V_{(-1,q_L)}(0; w_2, q_H) \\ &= A(\theta) \left[1 + \rho + \nu + \frac{2\sqrt{q_H + v}}{1 - \delta(1 - \pi)} - c(\underline{s}|q_L) \right] \geq 0, \end{aligned}$$

where $A(\theta)$ is a positive constant. Therefore, all signaling strategies in σ are optimal.

Finally, consider policy choices of incumbents. For low quality incumbents, $p(z, q_L) = z$ is obviously optimal since policy choices do not affect the election outcome. Now note that, for every $z \in (w_1, w_2)$, $\psi(z, w_1|\theta) = W_{(z,q_H)}(w_1; \sigma) - W_{(z,q_H)}(z; \sigma)$. Also, for every $z \in [w_2, 1]$, $\eta(z, w_1|\theta) = W_{(z,q_H)}(w_1; \sigma) - W_{(z,q_H)}(w_2; \sigma)$, $\mu(z, w_1|\theta) = W_{(z,q_H)}(w_1; \sigma) - W_{(z,q_H)}(z; \sigma)$, and $\gamma(z|\theta) = W_{(z,q_H)}(w_2; \sigma) - W_{(z,q_H)}(z; \sigma)$. Thus, by construction and symmetry, all policy choices of high quality incumbents are optimal. Therefore, σ is an equilibrium.

Proof of Proposition 1

For $t = 1, 2, \dots$, let P^t denote the *ex ante* probability that the election winner at t is the incumbent at $t - 1$ in the equilibrium σ in Theorem 1. Let L^t denote the probability that the election winner at t is of low quality. We will show that P^t is increasing and converges to one and that L^t is decreasing and converges to zero.

Let $T_1 = [-c_1, c_1] \times \{q_H\}$, $T_2 = ([-c_2, c_1] \cup (c_1, c_2]) \times \{q_H\}$, and $T_3 = (X \times Q) \setminus (T_1 \cup T_2)$. For $t = 0, 1, \dots$, let P_1^t be the probability that the type of office holder at t belongs to T_1 , P_2^t be the probability that it belongs to T_2 , and P_3^t be the probability that it belongs to T_3 . Let $N_1 = \int_{-c_1}^{c_1} f(y)dy$, $N_2 = \int_{-c_2}^{-c_1} f(y)dy + \int_{c_1}^{c_2} f(y)dy$, and $N_3 = \int_{-1}^{-c_2} f(y)dy + \int_{c_2}^1 f(y)dy$.

From the specification of strategies in σ ,

$$P_2^t = P_2^{t-1}(1 - \pi) + [P_2^{t-1} + P_3^{t-1}]\pi N_2, \quad (3)$$

and

$$P_3^t = P_3^{t-1}(1 - \pi) + [P_2^{t-1} + P_3^{t-1}]\pi N_3 \quad (4)$$

for every $t = 1, 2, \dots$. By adding (3) and (4), we obtain $P_2^t + P_3^t = [1 - \pi N_1](P_2^{t-1} + P_3^{t-1})$.

Since $P_2^0 + P_3^0 = 1 - \pi N_1$,

$$P_2^t + P_3^t = [1 - \pi N_1]^{t+1}, \quad (5)$$

implying $P_1^t = 1 - [1 - \pi N_1]^{t+1}$. From the strategies, $P^t = P_2^{t-1} + P_2^{t-1}(1 - \pi)$. Then, since $P_1^t \rightarrow 1$, $P^t \rightarrow 1$. Also, $L^t = P_3^{t-1}(1 - \pi)$. Since $P_2^t + P_3^t \rightarrow 0$, $L^t \rightarrow 0$.

Using (3), (4), and (5), we can obtain

$$P_2^t = \frac{N_2}{N_2 + N_3} \left[(1 - \pi N_1)^{t+1} - (1 - \pi)^{t+1} \right],$$

and

$$P_3^t = (1 - \pi)^{t+1} + \frac{N_3}{N_2 + N_3} \left[(1 - \pi N_1)^{t+1} - (1 - \pi)^{t+1} \right].$$

Then, for every $t = 1, 2, \dots$,

$$\begin{aligned} P^t - P^{t-1} &= (1 - \pi N_1)^{t-1} \pi N_1 \left[1 - \frac{N_2(1 - \pi)}{N_2 + N_3} \right] + (1 - \pi)^{t-1} \pi \frac{N_2(1 - \pi)}{N_2 + N_3} \\ &> 0. \end{aligned}$$

Thus, P^t is increasing. Also, for every $t = 1, 2, \dots$,

$$\begin{aligned} P_3^t - P_3^{t-1} &= -(1 - \pi)^t \pi \left[1 - \frac{N_3}{N_2 + N_3} \right] - (1 - \pi N_1)^t \pi N_1 \frac{N_3}{N_2 + N_3} \\ &< 0, \end{aligned}$$

implying P_3^t is decreasing. Therefore, L^t is decreasing.

Proof of Proposition 2

We will prove the proposition for the case that $c_1 < w_2 < c_2 < 1$. The proofs for the other cases are similar and simpler.

Simple exercises of calculus show that $\frac{\partial \gamma}{\partial \delta}(z|\theta) \geq 0$, $\frac{\partial \gamma}{\partial \rho}(z|\theta) \geq 0$, and $\frac{\partial \gamma}{\partial z}(z|\theta) \leq 0$. Also, $\frac{\partial}{\partial \delta}|_{x=w_1} \psi(z, x, \theta) \geq 0$, $\frac{\partial}{\partial \rho}|_{x=w_1} \psi(z, x, \theta) \geq 0$, $\frac{\partial}{\partial z}|_{x=w_1} \psi(z, x, \theta) \leq 0$, and $\frac{\partial}{\partial x}|_{x=w_1} \psi(z, x, \theta) \geq 0$. Then, by implicit function theory, $\frac{\partial \hat{c}_2}{\partial \delta}(\theta) \geq 0$, $\frac{\partial \hat{c}_2}{\partial \rho}(\theta) \geq 0$, $\frac{\partial}{\partial \delta}|_{x=w_1} \hat{c}_1(x|\theta) \geq 0$, $\frac{\partial}{\partial \rho}|_{x=w_1} \hat{c}_1(x|\theta) \geq 0$, and $\frac{\partial}{\partial x}|_{x=w_1} \hat{c}_1(x|\theta) \geq 0$. Then $\frac{\partial}{\partial \delta}|_{x=w_1} U(x|\theta) \geq 0$ and $\frac{\partial}{\partial \rho}|_{x=w_1} U(x|\theta) \geq 0$, which implies $\frac{\partial}{\partial \delta}|_{x=w_1} K(x|\theta) \leq 0$ and $\frac{\partial}{\partial \rho}|_{x=w_1} K(x|\theta) \leq 0$.

For a given θ , let

$$\begin{aligned} E(x) &= \int_0^x y^2 f(y) dy + x^2 \int_x^{\hat{c}_1(x|\theta)} f(y) dy + \alpha(\theta) \int_{\hat{c}_1(x|\theta)}^{w_2(\theta)} y^2 f(y) dy \\ &\quad + \alpha(\theta) w_2(\theta)^2 \int_{w_2(\theta)}^{\hat{c}_2(\theta)} f(y) dy + (1 - \delta) \int_{\hat{c}_2(\theta)}^1 y^2 f(y) dy \\ &\quad + \delta \beta(\theta) \nu \int_{\hat{c}_2(\theta)}^1 f(y) dy + \delta \beta(\theta) q_H \int_{\hat{c}_2(\theta)}^1 f(y) dy, \end{aligned}$$

and

$$D(x) = \int_0^{\hat{c}_1(x|\theta)} f(y) dy + \alpha(\theta) \int_{\hat{c}_1(x|\theta)}^1 f(y) dy.$$

Then, $U(x|\theta) = -\frac{E(x)}{D(x)} + q_H$, implying $K(x|\theta) = \frac{E(x)}{D(x)} - x^2$. Then,

$$\begin{aligned} \frac{\partial}{\partial x} \Big|_{x=w_1} K(x|\theta) &= \frac{E'(w_1)D(w_1) - E(w_1)D'(w_1)}{D(w_1)^2} - 2w_1 \\ &= \left(-\frac{E(w_1)}{D(w_1)} \right) \frac{D'(w_1)}{D(w_1)} + \frac{E'(w_1)}{D(w_1)} - 2w_1. \end{aligned}$$

Note that

$$D'(w_1) = f(\hat{c}_1(w_1|\theta)) \frac{\partial}{\partial x} \Big|_{x=w_1} \hat{c}_1(x|\theta) [1 - \alpha(\theta)] < 0.$$

Since $\frac{E(w_1)}{D(w_1)} = w_1^2 > 0$, $(-\frac{E(w_1)}{D(w_1)}) \frac{D'(w_1)}{D(w_1)} < 0$. Also,

$$E'(w_1) = [w_1^2 - \alpha(\theta) \hat{c}_1(w_1|\theta)^2] f(\hat{c}_1(w_1|\theta)) \frac{\partial}{\partial x} \Big|_{x=w_1} \hat{c}_1(x|\theta) + 2w_1 \int_{w_1}^{\hat{c}_1(w_1|\theta)} f(y) dy.$$

Note that

$$\frac{2w_1 \int_{w_1}^{\hat{c}_1(w_1|\theta)} f(y) dy}{D(w_1)} - 2w_1 = 2w_1 \left[\frac{\int_{w_1}^{\hat{c}_1(w_1|\theta)} f(y) dy}{D(w_1)} - 1 \right] < 0.$$

Since $w_1^2 - \alpha(\theta)\hat{c}_1(w_1|\theta)^2 < 0$, $\frac{E'(w_1)}{D(w_1)} - 2w_1 < 0$. Thus, $\frac{\partial}{\partial x}|_{x=w_1} K(x|\theta) < 0$. Therefore,

$$\frac{\partial w_1}{\partial \delta} = -\frac{\frac{\partial}{\partial \delta}|_{x=w_1} K(x|\theta)}{\frac{\partial}{\partial x}|_{x=w_1} K(x|\theta)} \leq 0 \text{ and } \frac{\partial w_1}{\partial \rho} = -\frac{\frac{\partial}{\partial \rho}|_{x=w_1} K(x|\theta)}{\frac{\partial}{\partial x}|_{x=w_1} K(x|\theta)} \leq 0,$$

proving the first part and the second part of the proposition. The remaining parts can be proven similarly.

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Abstract

Strategic Challengers and the Incumbency Advantage

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The existing formal studies on the incumbency advantage do not take strategic choices of electoral challengers into account. We develop a dynamic model of infinitely repeated electoral competition that incorporates strategic challengers and shows the endogenous emergence of the incumbency advantage. Politicians' characteristics are of two dimensions, policy and quality. We assume that policy preferences are heterogeneous among players but everyone prefers to elect a high quality candidate. Candidates' types which are defined by policy preference and quality are private information but by observing the incumbent's policy choice voters know the incumbent's type while nothing is revealed about a challenger. With a strategic challenger we find an equilibrium in which only high quality challengers campaign to distinguish themselves from low quality challengers and win the election. However, we still find a strong incumbency advantage accruing from voters' ignorance about the challenger's policy preference relative to their knowledge of the incumbent.

Key Words

Incumbency advantage, strategic challengers, US elections, electoral campaign, formal modeling of repeated elections